Numerical Data of Complex Capacitance and Conductance for the Circular Arc Rule in Dielectric Relaxations

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The analytical expressions of the complex capacitance and the complex conductance have been derived for the circular arc rule on dielectric relaxation. Numerical data of the normalized complex capacitance and conductance as a function of frequency are tabulated over the wide range of the parameter for the distribution of relaxation times to facilitate the analysis of data on dielectric relaxation. For general cases of the circular arc rule, neither the real nor the imaginary part of the complex conductance tend to zero or finite values but they increase infinitely at high frequencies and give complicated loci dissimilar to circular arcs in the complex plane.

I. INTRODUCTION

It is well known that the frequency dependence of the dielectric constants and the loss factors for a number of dielectric materials is represented satisfactorily by the so-called circular arc rule which was proposed by Cole and Cole.[1,2]

Some incorrect interpretations and treatments, however, are sometimes made for lack of the consideration on the complex conductivity associated with the dielectric relaxation which is given by the circular arc rule. For example Pauly and Packer[3] assumed that the conductivity tends to a finite value at high frequencies for the dielectric dispersion which is characterized by a circular arc. Such an incomplete consideration led them to a doubtful interpretation that the further increase in the conductivity at higher frequencies far beyond the critical frequency implies the beginning of another dielectric dispersion.

An incomplete analysis of the complex conductivity for the circular arc rule results in an incorrect deduction that the locus of the complex conductivity in the complex plane would also give a circular arc. Following such a deduction Pauly *et al.*[3,4] suggested a false relation that the critical frequency may be the geometical mean of loss maximum frequencies in the complex dielectric constant plane and the complex conductivity plane.

In the present paper, the analytical expressions of the complex dielectric constant for the circular arc rule and the corresponding complex conductivity are derived. Numerical values of normalized dielectric constants and normalized conductivities are calculated from these expressions to discuss the behavior in

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the complex planes. The interpretation and the treatment which were made in the references[3,4] for the frequency dependence of the conductivity are also discussed.

II. THE EXPRESSIONS OF THE COMPLEX CAPACITANCE AND THE COMPLEX CONDUCTANCE

In this paper complex capacitances and conductances are used instead of complex dielectric constants and conductivities for brevity.

A. The General Relations between the Relaxation Terms and the Measured Capacitance and Conductance

The complex capacitance C^* and the complex conductance G^* for the whole system observed are written in the form

$$C^* = \bar{C}' - j\bar{C}'' + C_\infty + \frac{1}{j\omega} G_0 \tag{1}$$

and

$$G^* = \bar{G}' + j\bar{G}'' + j\omega C_\infty + G_0 \tag{2}$$

where $\bar{C}'$ and $\bar{C}''$ are the real and the imaginary part of the complex capacitance associated with a dielectric dispersion, $\bar{G}'$ and $\bar{G}''$ the real and the imaginary part of the corresponding complex conductance, C_∞ the capacitance at infinite frequency, G_0 the dc conductance, ω the angular frequency, and $j=\sqrt{-1}$.

Since we have a relation

$$G^* = j\omega C^*, \tag{3}$$

it follows from Eqs. 1 and 2 that

$$\bar{G}'' = \omega \bar{C}' \tag{4}$$

and

$$\bar{G}' = \omega \bar{C}''. \tag{5}$$

The capacitance C' and the conductance G' measured are related to C^* and G^* with the relations

$$C^* = C' + \frac{1}{j\omega} G' \tag{6}$$

and

$$G^* = G' + j\omega C'. \tag{7}$$

Equating Eq. 6 to Eq. 1, we have

$$\bar{C}' = C' - C_\infty \tag{8}$$

and

$$\bar{C}'' = \frac{1}{\omega}(G' - G_0). \tag{9}$$

Insertion of Eqs. 8 and 9 into Eqs. 4 and 5 gives

$$\bar{G}' = G' - G_0 \tag{10}$$

and

$$\bar{G}''=\omega(C'-C_\infty). \tag{11}$$

Thus it is possible to evaluate $\bar{C}'$, $\bar{C}''$, $\bar{G}'$, and $\bar{G}''$ from the measured C' and G' by use of Eqs. 8 to 11.

B. The Expressions for the Circular Arc Rule

According to Cole and Cole[1] the circular arc rule is given by

$$\bar{C}'-j\bar{C}''=(C_0-C_\infty)\frac{1}{1+(j\omega\tau)^\beta} \quad ; \quad 1\geqslant\beta>0, \tag{12}$$

where C_0 is the static capacitance, τ the most probable relaxation time, and β denotes the parameter for the distribution of relaxation times.
If we put

$$\omega\tau=e^x, \tag{13}$$

then we have

$$(\omega\tau)^\beta=e^{\beta x} \tag{14}$$

and

$$x=\ln(\omega\tau). \tag{15}$$

If the right of Eq. 12 is separated into the real and the imaginary part, and if Eq. 14 is used to derive the expressions with respect to x, then the following equations are obtained.

$$\frac{\bar{C}'}{C_0-C_\infty}=\frac{C'-C_\infty}{C_0-C_\infty}=\frac{1+(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)}{1+2(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)+(\omega\tau)^{2\beta}}\quad, \tag{16}$$

$$=\frac{e^{-\beta x}+\cos\left(\frac{\pi}{2}\beta\right)}{e^{\beta x}+e^{-\beta x}+2\cos\left(\frac{\pi}{2}\beta\right)}\quad, \tag{17}$$

$$=\frac{1}{2}\left(1-\frac{\sinh\beta x}{\cosh\beta x+\cos\left(\frac{\pi}{2}\beta\right)}\right)\quad. \tag{18}$$

$$\frac{\bar{C}''}{C_0-C_\infty}=\frac{\frac{1}{\omega}(G'-G_0)}{C_0-C_\infty}=\frac{(\omega\tau)^\beta\sin\left(\frac{\pi}{2}\beta\right)}{1+2(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)+(\omega\tau)^{2\beta}}\quad, \tag{19}$$

$$=\frac{\sin\left(\frac{\pi}{2}\beta\right)}{e^{\beta x}+e^{-\beta x}+2\cos\left(\frac{\pi}{2}\beta\right)}\quad, \tag{20}$$

$$=\frac{\sin\left(\frac{\pi}{2}\beta\right)}{2\left[\cosh\beta x+\cos\left(\frac{\pi}{2}\beta\right)\right]}\quad. \tag{21}$$

By use of Eqs. 16 to 21 the equations for $\bar{G}'$ and $\bar{G}''$ are derived as follows.

$$\frac{\bar{G}'\tau}{C_0-C_\infty}=\frac{\bar{C}''\omega\tau}{C_0-C_\infty}=\frac{\omega\tau(\omega\tau)^\beta\sin\left(\frac{\pi}{2}\beta\right)}{1+2(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)+(\omega\tau)^{2\beta}}\quad, \tag{22}$$

$$=\frac{e^x\sin\left(\frac{\pi}{2}\beta\right)}{e^{\beta x}+e^{-\beta x}+2\cos\left(\frac{\pi}{2}\beta\right)}\quad, \tag{23}$$

$$=\frac{\sin\left(\frac{\pi}{2}\beta\right)}{2}\cdot\frac{\cosh x+\sinh x}{\cosh\beta x+\cos\left(\frac{\pi}{2}\beta\right)}\quad. \tag{24}$$

$$\frac{\bar{G}''\tau}{C_0-C_\infty}=\frac{\bar{C}'\omega\tau}{C_0-C_\infty}=\frac{\omega\tau\left[1+(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)\right]}{1+2(\omega\tau)^\beta\cos\left(\frac{\pi}{2}\beta\right)+(\omega\tau)^{2\beta}}\quad, \tag{25}$$

$$=\frac{e^x\left[e^{-\beta x}+\cos\left(\frac{\pi}{2}\beta\right)\right]}{e^{\beta x}+e^{-\beta x}+2\cos\left(\frac{\pi}{2}\beta\right)}\quad, \tag{26}$$

$$=\frac{\cosh x+\sinh x}{2}\left(1-\frac{\sinh\beta x}{\cosh\beta x+\cos\left(\frac{\pi}{2}\beta\right)}\right)\quad. \tag{27}$$

III. THE RESULTS OF NUMERICAL CALCULATION OF THE COMPLEX CAPACITANCE AND THE COMPLEX CONDUCTANCE

The values of $\frac{\bar{C}'}{C_0-C_\infty}$, $\frac{\bar{C}''}{C_0-C_\infty}$, $\frac{\bar{G}'\tau}{C_0-C_\infty}$, and $\frac{\bar{G}''\tau}{C_0-C_\infty}$ were calculated for respective values of β by use of Eqs. 16 to 27. In Table 1 the numerical values of $\frac{\bar{C}'}{C_0-C_\infty}$ and $\frac{\bar{C}''}{C_0-C_\infty}$ are given over the range of $\log_{10}(\omega\tau)$ of 0 to 3 for values of β of 1 to 0.05 in a floating point form. The five significant digits denote the mantissa in which the decimal point is just to the left of the most significant digit. One digit with a sign to the right of the mantissa denotes the exponent which specifies the position of the decimal point by a sign and the digit. Thus a number such as 0.012345 or 0.12345×10^{-1} is written in the form $12345-1$. Evidently Eq. 20 is an even function of x. Equation 17 gives an odd function of x with respect to a coordinate system whose origin is located at $\ln(\omega\tau)=0$ and $\frac{\bar{C}'}{C_0-C_\infty}=0.5$. Thus the values of $\bar{C}'$ and $\bar{C}''$ at $\ln(\omega\tau)<0$ are readily obtained from those at $\ln(\omega\tau)>0$ which are given in Table 1.

Numerical values of $\frac{\bar{G}'\tau}{C_0-C_\infty}$ and $\frac{\bar{G}''\tau}{C_0-C_\infty}$ are given in Table 2. The frequency dependence of the complex capacitance and conductance is shown graphically in Figs. 1, 2, 3, and 4 for several representative values of β. The plots of the values in the complex capacitance and conductance planes are shown in Figs. 5 and 6.

Table 1. Numerical Data of Real and Imaginary Part of the Complex Capacitance for the Circular Arc Rule. In the Table the numerical format designated as 12345−1 denotes the number 0.012345 or 0.12345×10^{-1}.

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=1.00$		$\beta=0.99$		$\beta=0.98$		$\beta=0.97$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	50000−0	50000−0	49221−0	50000−0	48453−0	50000−0	47698−0
0.1	1.2589	38686−0	48703−0	38965−0	47988−0	39235−0	47281−0	39497−0	46583−0
0.2	1.5849	28475−0	45129−0	28962−0	44580−0	29437−0	44032−0	29900−0	43485−0
0.3	1.9953	20076−0	40057−0	20669−0	39715−0	21252−0	39368−0	21826−0	39016−0
0.4	2.5119	13681−0	34364−0	14288−0	34218−0	14890−0	34062−0	15486−0	33896−0
0.5	3.1623	90909−1	28748−0	96508−1	28754−0	10211−0	28749−0	10771−0	28733−0
0.6	3.9811	59351−1	23628−0	64187−1	23736−0	69066−1	23834−0	73986−1	23923−0
0.7	5.0119	38287−1	19189−0	42292−1	19355−0	46366−1	19515−0	50506−1	19667−0
0.8	6.3096	24503−1	15461−0	27734−1	15654−0	31045−1	15844−0	34435−1	16028−0
0.9	7.9433	15602−1	12393−0	18165−1	12592−0	20811−1	12790−0	23539−1	12985−0
1.0	10.000	99010−2	99010−1	11916−1	10094−0	14008−1	10286−0	16180−1	10477−0
1.1	12.589	62700−2	78935−1	78450−2	80718−1	94909−2	82510−1	11209−1	84309−1
1.2	15.849	39653−2	62846−1	51937−2	64453−1	64848−2	66077−1	78406−2	67717−1
1.3	19.953	25056−2	49993−1	34632−2	51415−1	44752−2	52858−1	55435−2	54323−1
1.4	25.119	15824−2	39748−1	23293−2	40988−1	31228−2	42252−1	39648−2	43541−1
1.5	31.623	99900−3	31591−1	15824−2	32661−1	22053−2	33757−1	28695−2	34878−1
1.6	39.811	63056−3	25103−1	10869−2	26019−1	15766−2	26961−1	21014−2	27927−1
1.7	50.119	39795−3	19945−1	75562−3	20724−1	11413−2	21528−1	15564−2	22355−1
1.8	63.096	25113−3	15845−1	53196−3	16505−1	83618−3	17187−1	11652−2	17891−1
1.9	79.433	15847−3	12587−1	37935−3	13143−1	61974−3	13719−1	88090−3	14317−1
2.0	100.00	99990−4	99990−2	27400−3	10465−1	46426−3	10951−1	67187−3	11455−1
2.1	125.89	63091−4	79428−2	20038−3	83330−2	35118−3	87400−2	51644−3	91644−2
2.2	158.49	39810−4	63093−2	14827−3	66349−2	26794−3	69755−2	39966−3	73315−2
2.3	199.53	25118−4	50117−2	11091−3	52828−2	20598−3	55670−2	31108−3	58649−2
2.4	251.19	15849−4	39810−2	83775−4	42061−2	15938−3	44428−2	24331−3	46915−2
2.5	316.23	99990−5	31622−2	63824−4	33489−2	12401−3	35455−2	19110−3	37528−2
2.6	398.11	63075−5	25119−2	48997−4	26663−2	96935−4	28295−2	15060−3	30019−2
2.7	501.19	39805−5	19953−2	37855−4	21228−2	76062−4	22580−2	11902−3	24011−2
2.8	630.96	25120−5	15849−2	29409−4	16901−2	59879−4	18019−2	94270−4	19206−2
2.9	794.33	15850−5	12589−2	22949−4	13456−2	47258−4	14380−2	74813−4	15362−2
3.0	1000.0	10000−5	10000−2	17978−4	10713−2	37380−4	11475−2	59461−4	12288−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.96$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.96$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.95$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.95$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.94$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.94$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.93$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.93$ $\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	46953−0	50000−0	46220−0	50000−0	45496−0	50000−0	44784−0
0.1	1.2589	39752−0	45894−0	40000−0	45213−0	40241−0	44539−0	40475−0	43874−0
0.2	1.5849	30353−0	42940−0	30795−0	42397−0	31227−0	41856−0	31648−0	41317−0
0.3	1.9953	22389−0	38658−0	22943−0	38295−0	23487−0	37928−0	24022−0	37557−0
0.4	2.5119	16077−0	33721−0	16663−0	33537−0	17243−0	33343−0	17817−0	33142−0
0.5	3.1623	11330−0	28706−0	11889−0	28670−0	12447−0	28622−0	13004−0	28565−0
0.6	3.9811	78943−1	24002−0	83936−1	24072−0	88961−1	24132−0	94016−1	24182−0
0.7	5.0119	54712−1	19813−0	58982−1	19950−0	63312−1	20080−0	67702−1	20203−0
0.8	6.3096	37904−1	16208−0	41452−1	16383−0	45077−1	16552−0	48778−1	16716−0
0.9	7.9433	26349−1	13177−0	29243−1	13367−0	32220−1	13554−0	35281−1	13737−0
1.0	10.000	18432−1	10668−0	20765−1	10858−0	23181−1	11046−0	25680−1	11233−0
1.1	12.589	13002−1	86113−1	14871−1	87921−1	16816−1	89730−1	18841−1	91540−1
1.2	15.849	92628−2	69372−1	10753−1	71040−1	12314−1	72720−1	13947−1	74411−1
1.3	19.953	66702−2	55808−1	78572−2	57313−1	91065−2	58837−1	10420−1	60379−1
1.4	25.119	48573−2	44853−1	58023−2	46189−1	68017−2	47548−1	78577−2	48930−1
1.5	31.623	35770−2	36024−1	43297−2	37196−1	51295−2	38393−1	59785−2	39614−1
1.6	39.811	26629−2	28920−1	32630−2	29937−1	39036−2	30981−1	45866−2	32050−1
1.7	50.119	20027−2	23208−1	24818−2	24085−1	29955−2	24988−1	35455−2	25917−1
1.8	63.096	15205−2	18620−1	19035−2	19372−1	23158−2	20148−1	27592−2	20949−1
1.9	79.433	11641−2	14936−1	14708−2	15577−1	18023−2	16241−1	21602−2	16928−1
2.0	100.00	89797−3	11979−1	11438−2	12524−1	14106−2	13089−1	16998−2	13676−1
2.1	125.89	69718−3	96068−2	89449−3	10068−1	11095−2	10548−1	13435−2	11047−1
2.2	158.49	54431−3	77036−2	70287−3	80924−2	87633−3	84984−2	10658−2	89223−2
2.3	199.53	42697−3	61771−2	55451−3	65042−2	69457−3	68468−2	84811−3	72053−2
2.4	251.19	33625−3	49529−2	43893−3	52274−2	55212−3	55157−2	67666−3	58183−2
2.5	316.23	26568−3	39712−2	34840−3	42011−2	43994−3	44432−2	54102−3	46980−2
2.6	398.11	21049−3	31839−2	27718−3	33762−2	35125−3	35791−2	43334−3	37931−2
2.7	501.19	16714−3	25527−2	22094−3	27131−2	28090−3	28829−2	34760−3	30625−2
2.8	630.96	13296−3	20466−2	17638−3	21803−2	22494−3	23221−2	27915−3	24725−2
2.9	794.33	10593−3	16408−2	14098−3	17520−2	18033−3	18703−2	22440−3	19961−2
3.0	1000.0	84498−4	13154−2	11280−3	14079−2	14469−3	15064−2	18053−3	16114−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.92$		$\beta=0.91$		$\beta=0.90$		$\beta=0.89$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000-0	44081-0	50000-0	43388-0	50000-0	42704-0	50000-0	42029-0
0.1	1.2589	40703-0	43216-0	40925-0	42566-0	41141-0	41923-0	41351-0	41287-0
0.2	1.5849	32060-0	40780-0	32462-0	40245-0	32854-0	39712-0	33238-0	39182-0
0.3	1.9953	24547-0	37181-0	25063-0	36802-0	25570-0	36419-0	26069-0	36033-0
0.4	2.5119	18385-0	32931-0	18947-0	32713-0	19503-0	32487-0	20053-0	32253-0
0.5	3.1623	13560-0	28497-0	14113-0	28420-0	14665-0	28332-0	15215-0	28236-0
0.6	3.9811	99098-1	24223-0	10420-0	24254-0	10933-0	24276-0	11448-0	24287-0
0.7	5.0119	72149-1	20317-0	76652-1	20423-0	81207-1	20521-0	85813-1	20610-0
0.8	6.3096	52555-1	16875-0	56405-1	17027-0	60329-1	17173-0	64324-1	17312-0
0.9	7.9433	38424-1	13917-0	41651-1	14092-0	44960-1	14264-0	48352-1	14432-0
1.0	10.000	28263-1	11419-0	30930-1	11602-0	33682-1	11783-0	36521-1	11962-0
1.1	12.589	20945-1	93348-1	23131-1	95152-1	25400-1	96950-1	27753-1	98740-1
1.2	15.849	15653-1	76111-1	17435-1	77819-1	19294-1	79532-1	21233-1	81250-1
1.3	19.953	11800-1	61938-1	13248-1	63512-1	14767-1	65101-1	16359-1	66704-1
1.4	25.119	89724-2	50333-1	10148-1	51757-1	11387-1	53202-1	12690-1	54666-1
1.5	31.623	68787-2	40860-1	78324-2	42131-1	88417-2	43425-1	99089-2	44743-1
1.6	39.811	53140-2	33145-1	60879-2	34266-1	69104-2	35412-1	77838-2	36584-1
1.7	50.119	41337-2	26871-1	47621-2	27851-1	54328-2	28857-1	61478-2	29890-1
1.8	63.096	32354-2	21775-1	37461-2	22626-1	42933-2	23503-1	48790-2	24406-1
1.9	79.433	25460-2	17639-1	29615-2	18375-1	34083-2	19134-1	38884-2	19919-1
2.0	100.00	20128-2	14286-1	23511-2	14918-1	27164-2	15573-1	31101-2	16252-1
2.1	125.89	15976-2	11567-1	18734-2	12108-1	21721-2	12671-1	24954-2	13255-1
2.2	158.49	12724-2	93647-2	14973-2	98262-2	17418-2	10307-1	20074-2	10809-1
2.3	199.53	10162-2	75806-2	11998-2	79732-2	14001-2	83837-2	16184-2	88127-2
2.4	251.19	81344-3	61358-2	96343-3	64688-2	11276-2	68180-2	13071-2	71841-2
2.5	316.23	65244-3	49660-2	77503-3	52479-2	90969-3	55442-2	10574-2	58557-2
2.6	398.11	52414-3	40190-2	62439-3	42571-2	73487-3	45081-2	85645-3	47725-2
2.7	501.19	42162-3	32524-2	50363-3	34531-2	59431-3	36653-2	69443-3	38894-2
2.8	630.96	33953-3	26319-2	40664-3	28009-2	48108-3	29799-2	56354-3	31696-2
2.9	794.33	27366-3	21297-2	32859-3	22718-2	38972-3	24226-2	45765-3	25828-2
3.0	1000.0	22073-3	17233-2	26570-3	18425-2	31591-3	19695-2	37187-3	21046-2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.88$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.88$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.87$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.87$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.86$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.86$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.85$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.85$ $\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	41364−0	50000−0	40706−0	50000−0	40058−0	50000−0	39417−0
0.1	1.2589	41555−0	40658−0	41755−0	40036−0	41949−0	39421−0	42138−0	38812−0
0.2	1.5849	33613−0	38654−0	33979−0	38128−0	34337−0	37605−0	34687−0	37084−0
0.3	1.9953	26558−0	35644−0	27039−0	35252−0	27511−0	34858−0	27975−0	34461−0
0.4	2.5119	20597−0	32013−0	21135−0	31765−0	21666−0	31510−0	22191−0	31248−0
0.5	3.1623	15763−0	28129−0	16308−0	28013−0	16851−0	27888−0	17391−0	27755−0
0.6	3.9811	11965−0	24289−0	12483−0	24282−0	13002−0	24264−0	13522−0	24237−0
0.7	5.0119	90468−1	20691−0	95170−1	20764−0	99915−1	20827−0	10470−0	20882−0
0.8	6.3096	68389−1	17445−0	72523−1	17571−0	76723−1	17690−0	80990−1	17802−0
0.9	7.9433	51825−1	14594−0	55381−1	14752−0	59017−1	14905−0	62733−1	15053−0
1.0	10.000	39445−1	12138−0	42457−1	12312−0	45555−1	12482−0	48741−1	12649−0
1.1	12.589	30191−1	10052−0	32715−1	10229−0	35327−1	10404−0	38028−1	10577−0
1.2	15.849	23252−1	82971−1	25354−1	84692−1	27541−1	86411−1	29813−1	88127−1
1.3	19.953	18025−1	68318−1	19767−1	69943−1	21589−1	71577−1	23491−1	73218−1
1.4	25.119	14062−1	56149−1	15503−1	57650−1	17016−1	59167−1	18603−1	60699−1
1.5	31.623	11036−1	46083−1	12226−1	47446−1	13481−1	48830−1	14803−1	50235−1
1.6	39.811	87103−2	37781−1	96922−2	39003−1	10732−1	40249−1	11832−1	41520−1
1.7	50.119	69093−2	30948−1	77195−2	32033−1	85809−2	33145−1	94958−2	34282−1
1.8	63.096	55051−2	25335−1	61739−2	26291−1	68876−2	27273−1	76484−2	28282−1
1.9	79.433	44035−2	20730−1	49558−2	21566−1	55472−2	22429−1	61800−2	23318−1
2.0	100.00	35342−2	16955−1	39905−2	17683−1	44808−2	18436−1	50073−2	19215−1
2.1	125.89	28447−2	13863−1	32219−2	14493−1	36286−2	15148−1	40668−2	15827−1
2.2	158.49	22953−2	11332−1	26073−2	11876−1	29448−2	12443−1	33096−2	13032−1
2.3	199.53	18559−2	92611−2	21140−2	97294−2	23942−2	10218−1	26980−2	10728−1
2.4	251.19	15031−2	75676−2	17168−2	79692−2	19495−2	83897−2	22027−2	88298−2
2.5	316.23	12192−2	61829−2	13961−2	65266−2	15895−2	68873−2	18004−2	72659−2
2.6	398.11	99005−3	50511−2	11366−2	53445−2	12973−2	56533−2	14731−2	59781−2
2.7	501.19	80478−3	41262−2	92625−3	43761−2	10598−2	46398−2	12064−2	49181−2
2.8	630.96	65471−3	33703−2	75539−3	35829−2	86638−3	38077−2	98859−3	40456−2
2.9	794.33	53299−3	27528−2	61643−3	29333−2	70871−3	31246−2	81062−3	33276−2
3.0	1000.0	43414−3	22483−2	50330−3	24013−2	58003−3	25640−2	66500−3	27369−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.84$		$\beta=0.83$		$\beta=0.82$		$\beta=0.81$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	38784−0	50000−0	38159−0	50000−0	37541−0	50000−0	36931−0
0.1	1.2589	42322−0	38210−0	42502−0	37614−0	42677−0	37024−0	42848−0	36439−0
0.2	1.5849	35029−0	36566−0	35364−0	36050−0	35691−0	35536−0	36011−0	35026−0
0.3	1.9953	28430−0	34062−0	28878−0	33660−0	29317−0	33257−0	29749−0	32852−0
0.4	2.5119	22710−0	30981−0	23223−0	30707−0	23729−0	30427−0	24229−0	30142−0
0.5	3.1623	17928−0	27612−0	18462−0	27461−0	18992−0	27301−0	19519−0	27133−0
0.6	3.9811	14043−0	24201−0	14564−0	24155−0	15085−0	24100−0	15607−0	24035−0
0.7	5.0119	10953−0	20928−0	11439−0	20965−0	11929−0	20993−0	12422−0	21012−0
0.8	6.3096	85319−1	17906−0	89711−1	18003−0	94162−1	18092−0	98672−1	18173−0
0.9	7.9433	66528−1	15195−0	70402−1	15331−0	74353−1	15461−0	78380−1	15585−0
1.0	10.000	52013−1	12812−0	55373−1	12971−0	58821−1	13125−0	62355−1	13276−0
1.1	12.589	40818−1	10749−0	43698−1	10918−0	46669−1	11085−0	49732−1	11249−0
1.2	15.849	32172−1	89838−1	34621−1	91540−1	37160−1	93232−1	39790−1	94912−1
1.3	19.953	25476−1	74865−1	27545−1	76515−1	29702−1	78167−1	31947−1	79819−1
1.4	25.119	20267−1	62245−1	22010−1	63803−1	23834−1	65372−1	25742−1	66951−1
1.5	31.623	16195−1	51660−1	17659−1	53103−1	19198−1	54564−1	20815−1	56042−1
1.6	39.811	12995−1	42815−1	14223−1	44132−1	15519−1	45472−1	16886−1	46833−1
1.7	50.119	10467−1	35445−1	11496−1	36634−1	12587−1	37848−1	13741−1	39087−1
1.8	63.096	84588−2	29318−1	93212−2	30381−1	10238−1	31471−1	11213−1	32587−1
1.9	79.433	68564−2	24234−1	75789−2	25177−1	83498−2	26147−1	91718−2	27145−1
2.0	100.00	55720−2	20020−1	61773−2	20851−1	68253−2	21710−1	75186−2	22596−1
2.1	125.89	45384−2	16531−1	50455−2	17261−1	55902−2	18017−1	61749−2	18799−1
2.2	158.49	37035−2	13646−1	41285−2	14283−1	45865−2	14945−1	50796−2	15633−1
2.3	199.53	30272−2	11261−1	33834−2	11815−1	37684−2	12393−1	41844−2	12996−1
2.4	251.19	24777−2	92901−2	27763−2	97714−2	31001−2	10274−1	34510−2	10800−1
2.5	316.23	20304−2	76630−2	22807−2	80793−2	25530−2	85156−2	28490−2	89727−2
2.6	398.11	16654−2	63198−2	18753−2	66791−2	21044−2	70566−2	23540−2	74532−2
2.7	501.19	13671−2	52115−2	15432−2	55208−2	17358−2	58467−2	19464−2	61900−2
2.8	630.96	11230−2	42970−2	12707−2	45628−2	14327−2	48436−2	16103−2	51401−2
2.9	794.33	92303−3	35427−2	10469−2	37707−2	11832−2	40122−2	13330−2	42679−2
3.0	1000.0	75900−3	29207−2	86287−3	31159−2	97749−3	33232−2	11038−2	35433−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.80$		$\beta=0.79$		$\beta=0.78$		$\beta=0.77$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	36327−0	50000−0	35731−0	50000−0	35141−0	50000−0	34557−0
0.1	1.2589	43015−0	35861−0	43178−0	35288−0	43337−0	34721−0	43492−0	34159−0
0.2	1.5849	36324−0	34517−0	36631−0	34012−0	36931−0	33508−0	37224−0	33008−0
0.3	1.9953	30173−0	32445−0	30590−0	32037−0	30999−0	31628−0	31401−0	31217−0
0.4	2.5119	24722−0	29851−0	25210−0	29556−0	25691−0	29255−0	26166−0	28949−0
0.5	3.1623	20043−0	26957−0	20563−0	26773−0	21079−0	26582−0	21592−0	26383−0
0.6	3.9811	16128−0	23961−0	16649−0	23878−0	17170−0	23786−0	17689−0	23686−0
0.7	5.0119	12918−0	21021−0	13417−0	21022−0	13918−0	21013−0	14422−0	20996−0
0.8	6.3096	10324−0	18246−0	10786−0	18311−0	11253−0	18368−0	11725−0	18416−0
0.9	7.9433	82482−1	15703−0	86657−1	15813−0	90905−1	15917−0	95223−1	16014−0
1.0	10.000	65975−1	13421−0	69682−1	13562−0	73474−1	13697−0	77352−1	13827−0
1.1	12.589	52887−1	11410−0	56134−1	11567−0	59474−1	11721−0	62908−1	11871−0
1.2	15.849	42514−1	96576−1	45331−1	98222−1	48245−1	99848−1	51254−1	10145−0
1.3	19.953	34283−1	81468−1	36711−1	83112−1	39233−1	84749−1	41852−1	86377−1
1.4	25.119	27736−1	68537−1	29818−1	70129−1	31991−1	71725−1	34256−1	73323−1
1.5	31.623	22511−1	57535−1	24290−1	59042−1	26155−1	60561−1	28107−1	62090−1
1.6	39.811	18327−1	48215−1	19843−1	49617−1	21439−1	51038−1	23117−1	52475−1
1.7	50.119	14962−1	40350−1	16253−1	41637−1	17616−1	42946−1	19055−1	44278−1
1.8	63.096	12247−1	33729−1	13344−1	34898−1	14507−1	36093−1	15739−1	37313−1
1.9	79.433	10047−1	28170−1	10980−1	29223−1	11971−1	30303−1	13025−1	31410−1
2.0	100.00	82597−2	23510−1	90512−2	24451−1	98960−2	25420−1	10797−1	26418−1
2.1	125.89	68020−2	19609−1	74740−2	20446−1	81934−2	21310−1	89632−2	22203−1
2.2	158.49	56101−2	16347−1	61805−2	17087−1	67931−2	17855−1	74505−2	18650−1
2.3	199.53	46332−2	13622−1	51172−2	14275−1	56387−2	14953−1	62001−2	15658−1
2.4	251.19	38307−2	11348−1	42415−2	11921−1	46853−2	12518−1	51645−2	13140−1
2.5	316.23	31703−2	94512−2	35188−2	99520−2	38965−2	10476−1	43055−2	11024−1
2.6	398.11	26258−2	78695−2	29215−2	83065−2	32429−2	87649−2	35920−2	92455−2
2.7	501.19	21763−2	65514−2	24272−2	69317−2	27006−2	73318−2	29985−2	77524−2
2.8	630.96	18048−2	54532−2	20177−2	57835−2	22503−2	61319−2	25043−2	64992−2
2.9	794.33	14975−2	45385−2	16780−2	48248−2	18759−2	51276−2	20926−2	54477−2
3.0	1000.0	12430−2	37769−2	13961−2	40246−2	15643−2	42873−2	17491−2	45657−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.76$		$\beta=0.75$		$\beta=0.74$		$\beta=0.73$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	33980−0	50000−0	33409−0	50000−0	32844−0	50000−0	32285−0
0.1	1.2589	43643−0	33603−0	43791−0	33052−0	43936−0	32505−0	44077−0	31964−0
0.2	1.5849	37511−0	32510−0	37792−0	32014−0	38068−0	31521−0	38337−0	31031−0
0.3	1.9953	31796−0	30805−0	32184−0	30392−0	32566−0	29979−0	32940−0	29565−0
0.4	2.5119	26635−0	28639−0	27097−0	28324−0	27554−0	28005−0	28005−0	27682−0
0.5	3.1623	22100−0	26176−0	22605−0	25963−0	23105−0	25743−0	23602−0	25516−0
0.6	3.9811	18208−0	23576−0	18725−0	23457−0	19241−0	23330−0	19756−0	23195−0
0.7	5.0119	14927−0	20969−0	15435−0	20933−0	15944−0	20888−0	16454−0	20833−0
0.8	6.3096	12202−0	18455−0	12683−0	18486−0	13169−0	18508−0	13659−0	18522−0
0.9	7.9433	99611−1	16103−0	10407−0	16185−0	10859−0	16259−0	11317−0	16325−0
1.0	10.000	81313−1	13951−0	85358−1	14069−0	89485−1	14181−0	93694−1	14286−0
1.1	12.589	66435−1	12017−0	70055−1	12159−0	73768−1	12296−0	77575−1	12428−0
1.2	15.849	54361−1	10303−0	57567−1	10458−0	60871−1	10609−0	64275−1	10757−0
1.3	19.953	44568−1	87993−1	47383−1	89594−1	50299−1	91177−1	53317−1	92740−1
1.4	25.119	36616−1	74920−1	39074−1	76514−1	41631−1	78103−1	44290−1	79684−1
1.5	31.623	30150−1	63629−1	32286−1	65174−1	34518−1	66724−1	36848−1	68277−1
1.6	39.811	24879−1	53929−1	26730−1	55397−1	28671−1	56878−1	30705−1	58371−1
1.7	50.119	20572−1	45632−1	22171−1	47005−1	23855−1	48397−1	25626−1	49808−1
1.8	63.096	17043−1	38558−1	18423−1	39827−1	19880−1	41120−1	21419−1	42435−1
1.9	79.433	14145−1	32545−1	15333−1	33706−1	16592−1	34893−1	17928−1	36106−1
2.0	100.00	11757−1	27444−1	12779−1	28497−1	13867−1	29579−1	15024−1	30689−1
2.1	125.89	97862−2	23125−1	10665−1	24075−1	11604−1	25053−1	12605−1	26061−1
2.2	158.49	81556−2	19473−1	89112−2	20325−1	97204−2	21205−1	10586−1	22115−1
2.3	199.53	68040−2	16390−1	74531−2	17149−1	81504−2	17937−1	88988−2	18754−1
2.4	251.19	56816−2	13788−1	62391−2	14463−1	68396−2	15166−1	74862−2	15896−1
2.5	316.23	47481−2	11596−1	52267−2	12194−1	57438−2	12817−1	63021−2	13468−1
2.6	398.11	39708−2	97491−2	43815−2	10277−1	48265−2	10829−1	53084−2	11406−1
2.7	501.19	33226−2	81944−2	36750−2	86587−2	40579−2	91460−2	44737−2	96573−2
2.8	630.96	27816−2	68862−2	30839−2	72938−2	34133−2	77229−2	37719−2	81744−2
2.9	794.33	23297−2	57859−2	25890−2	61430−2	28722−2	65200−2	31815−2	69177−2
3.0	1000.0	19519−2	48606−2	21742−2	51729−2	24177−2	55035−2	26843−2	58532−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.72$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.72$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.71$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.71$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.70$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.70$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.69$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.69$ $\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	31731−0	50000−0	31183−0	50000−0	30640−0	50000−0	30102−0
0.1	1.2589	44215−0	31427−0	44350−0	30895−0	44483−0	30368−0	44612−0	29845−0
0.2	1.5849	38602−0	30543−0	38860−0	30057−0	39114−0	29574−0	39363−0	29093−0
0.3	1.9953	33308−0	29150−0	33670−0	28734−0	34026−0	28318−0	34375−0	27902−0
0.4	2.5119	28449−0	27355−0	28888−0	27025−0	29321−0	26691−0	29748−0	26353−0
0.5	3.1623	24094−0	25282−0	24581−0	25042−0	25065−0	24796−0	25544−0	24544−0
0.6	3.9811	20268−0	23051−0	20780−0	22899−0	21289−0	22739−0	21796−0	22571−0
0.7	5.0119	16966−0	20770−0	17479−0	20698−0	17992−0	20616−0	18506−0	20526−0
0.8	6.3096	14153−0	18526−0	14650−0	18522−0	15151−0	18508−0	15655−0	18485−0
0.9	7.9433	11782−0	16384−0	12253−0	16434−0	12729−0	16475−0	13211−0	16508−0
1.0	10.000	97982−1	14385−0	10235−0	14476−0	10679−0	14560−0	11132−0	14637−0
1.1	12.589	81474−1	12554−0	85465−1	12675−0	89548−1	12790−0	93722−1	12899−0
1.2	15.849	67780−1	10902−0	71385−1	11042−0	75091−1	11178−0	78898−1	11310−0
1.3	19.953	56439−1	94280−1	59665−1	95794−1	62998−1	97279−1	66437−1	98731−1
1.4	25.119	47052−1	81255−1	49920−1	82813−1	52894−1	84355−1	55978−1	85878−1
1.5	31.623	39279−1	69831−1	41814−1	71383−1	44455−1	72930−1	47204−1	74470−1
1.6	39.811	32837−1	59872−1	35068−1	61381−1	37402−1	62896−1	39841−1	64413−1
1.7	50.119	27489−1	51234−1	29447−1	52675−1	31503−1	54129−1	33660−1	55595−1
1.8	63.096	23044−1	43771−1	24757−1	45128−1	26563−1	46504−1	28465−1	47897−1
1.9	79.433	19342−1	37345−1	20838−1	38608−1	22421−1	39894−1	24094−1	41203−1
2.0	100.00	16253−1	31826−1	17559−1	32991−1	18944−1	34182−1	20413−1	35399−1
2.1	125.89	13673−1	27098−1	14810−1	28163−1	16020−1	29257−1	17308−1	30380−1
2.2	158.49	11512−1	23054−1	12502−1	24023−1	13559−1	25021−1	14686−1	26049−1
2.3	199.53	97016−2	19601−1	10562−1	20476−1	11484−1	21382−1	12470−1	22318−1
2.4	251.19	81817−2	16655−1	89295−2	17443−1	97328−2	18261−1	10595−1	19109−1
2.5	316.23	69044−2	14146−1	75538−2	14853−1	82536−2	15588−1	90070−2	16353−1
2.6	398.11	58298−2	12010−1	63935−2	12641−1	70027−2	13300−1	76604−2	13988−1
2.7	501.19	49249−2	10193−1	54140−2	10755−1	59440−2	11344−1	65179−2	11960−1
2.8	630.96	41621−2	86492−2	45864−2	91482−2	50473−2	96722−2	55477−2	10222−1
2.9	794.33	35188−2	73371−2	38866−2	77792−2	42873−2	82448−2	47235−2	87349−2
3.0	1000.0	29759−2	62229−2	32946−2	66137−2	36428−2	70264−2	40228−2	74621−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.68$		$\beta=0.67$		$\beta=0.66$		$\beta=0.65$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	29570−0	50000−0	29042−0	50000−0	28519−0	50000−0	28001−0
0.1	1.2589	44739−0	29327−0	44862−0	28813−0	44984−0	28303−0	45103−0	27796−0
0.2	1.5849	39606−0	28615−0	39845−0	28140−0	40080−0	27666−0	40309−0	27195−0
0.3	1.9953	34719−0	27486−0	35057−0	27069−0	35389−0	26652−0	35716−0	26235−0
0.4	2.5119	30169−0	26013−0	30585−0	25669−0	30995−0	25322−0	31399−0	24973−0
0.5	3.1623	26019−0	24285−0	26489−0	24022−0	26955−0	23753−0	27416−0	23478−0
0.6	3.9811	22300−0	22395−0	22803−0	22212−0	23303−0	22021−0	23800−0	21823−0
0.7	5.0119	19020−0	20427−0	19534−0	20319−0	20049−0	20203−0	20563−0	20077−0
0.8	6.3096	16163−0	18454−0	16672−0	18413−0	17185−0	18363−0	17699−0	18304−0
0.9	7.9433	13699−0	16533−0	14192−0	16548−0	14689−0	16555−0	15192−0	16553−0
1.0	10.000	11591−0	14706−0	12058−0	14767−0	12532−0	14821−0	13013−0	14865−0
1.1	12.589	97986−1	13002−0	10234−0	13098−0	10678−0	13187−0	11131−0	13269−0
1.2	15.849	82806−1	11436−0	86815−1	11557−0	90925−1	11673−0	95136−1	11782−0
1.3	19.953	69985−1	10015−0	73640−1	10153−0	77406−1	10286−0	81281−1	10415−0
1.4	25.119	59173−1	87379−1	62481−1	88855−1	65902−1	90302−1	69439−1	91718−1
1.5	31.623	50064−1	76000−1	53037−1	77518−1	56126−1	79019−1	59333−1	80502−1
1.6	39.811	42389−1	65931−1	45049−1	67446−1	47823−1	68957−1	50715−1	70460−1
1.7	50.119	35921−1	57069−1	38291−1	58550−1	40772−1	60036−1	43368−1	61524−1
1.8	63.096	30466−1	49307−1	32570−1	50730−1	34782−1	52166−1	37105−1	53613−1
1.9	79.433	25861−1	42533−1	27725−1	43883−1	29691−1	45252−1	31763−1	46637−1
2.0	100.00	21969−1	36641−1	23617−1	37908−1	25361−1	39198−1	27206−1	40510−1
2.1	125.89	18677−1	31530−1	20131−1	32708−1	21676−1	33913−1	23314−1	35143−1
2.2	158.49	15889−1	27107−1	17171−1	28193−1	18536−1	29310−1	19989−1	30454−1
2.3	199.53	13526−1	23285−1	14654−1	24282−1	15860−1	25309−1	17147−1	26367−1
2.4	251.19	11521−1	19988−1	12513−1	20897−1	13576−1	21838−1	14715−1	22810−1
2.5	316.23	98177−2	17148−1	10690−1	17974−1	11626−1	18831−1	12633−1	19720−1
2.6	398.11	83702−2	14705−1	91356−2	15451−1	99605−2	16229−1	10849−1	17038−1
2.7	501.19	71388−2	12604−1	78104−2	13277−1	85362−2	13980−1	93201−2	14714−1
2.8	630.96	60908−2	10799−1	66796−2	11404−1	73178−2	12038−1	80089−2	12701−1
2.9	794.33	51981−2	92505−2	57142−2	97926−2	62749−2	10362−1	68839−2	10960−1
3.0	1000.0	44375−2	79217−2	48895−2	84064−2	53820−2	89170−2	59183−2	94547−2

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.64$		$\beta=0.63$		$\beta=0.62$		$\beta=0.61$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	27488−0	50000−0	26979−0	50000−0	26474−0	50000−0	25973−0
0.1	1.2589	45219−0	27294−0	45333−0	26796−0	45445−0	26302−0	45554−0	25811−0
0.2	1.5849	40535−0	26726−0	40756−0	26260−0	40973−0	25796−0	41187−0	25334−0
0.3	1.9953	36037−0	25818−0	36353−0	25401−0	36663−0	24984−0	36969−0	24568−0
0.4	2.5119	31798−0	24621−0	32192−0	24266−0	32580−0	23910−0	32963−0	23550−0
0.5	3.1623	27873−0	23199−0	28325−0	22914−0	28773−0	22625−0	29217−0	22330−0
0.6	3.9811	24294−0	21618−0	24786−0	21405−0	25275−0	21186−0	25761−0	20961−0
0.7	5.0119	21076−0	19944−0	21590−0	19802−0	22102−0	19652−0	22613−0	19493−0
0.8	6.3096	18216−0	18235−0	18735−0	18158−0	19255−0	18072−0	19777−0	17977−0
0.9	7.9433	15699−0	16542−0	16210−0	16522−0	16726−0	16492−0	17245−0	16454−0
1.0	10.000	13500−0	14902−0	13994−0	14930−0	14495−0	14949−0	15002−0	14959−0
1.1	12.589	11592−0	13344−0	12062−0	13410−0	12540−0	13469−0	13027−0	13520−0
1.2	15.849	99447−1	11886−0	10386−0	11983−0	10837−0	12073−0	11297−0	12156−0
1.3	19.953	85266−1	10540−0	89362−1	10659−0	93568−1	10772−0	97885−1	10880−0
1.4	25.119	73092−1	93099−1	76863−1	94441−1	80753−1	95741−1	84762−1	96995−1
1.5	31.623	62659−1	81962−1	66107−1	83397−1	69678−1	84802−1	73375−1	86175−1
1.6	39.811	53726−1	71953−1	56861−1	73432−1	60121−1	74893−1	63508−1	76334−1
1.7	50.119	46083−1	63012−1	48919−1	64496−1	51880−1	65974−1	54969−1	67442−1
1.8	63.096	39543−1	55067−1	42099−1	56527−1	44778−1	57990−1	47583−1	59453−1
1.9	79.433	33946−1	48038−1	36243−1	49452−1	38659−1	50877−1	41197−1	52310−1
2.0	100.00	29155−1	41842−1	31213−1	43194−1	33386−1	44563−1	35677−1	45948−1
2.1	125.89	25051−1	36398−1	26892−1	37677−1	28841−1	38979−1	30903−1	40301−1
2.2	158.49	21535−1	31627−1	23178−1	32827−1	24923−1	34053−1	26776−1	35304−1
2.3	199.53	18520−1	27455−1	19984−1	28572−1	21544−1	29719−1	23206−1	30894−1
2.4	251.19	15933−1	23813−1	17236−1	24847−1	18629−1	25913−1	20117−1	27009−1
2.5	316.23	13713−1	20640−1	14871−1	21592−1	16113−1	22577−1	17443−1	23594−1
2.6	398.11	11805−1	17879−1	12834−1	18752−1	13940−1	19657−1	15128−1	20596−1
2.7	501.19	10166−1	15479−1	11079−1	16276−1	12063−1	17106−1	13123−1	17968−1
2.8	630.96	87570−2	13395−1	95662−2	14120−1	10441−1	14877−1	11386−1	15667−1
2.9	794.33	75449−2	11588−1	82618−2	12245−1	90390−2	12934−1	98810−2	13655−1
3.0	1000.0	65019−2	10021−1	71366−2	10615−1	78265−2	11240−1	85760−2	11896−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.60$		$\beta=0.59$		$\beta=0.58$		$\beta=0.57$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	25476−0	50000−0	24984−0	50000−0	24495−0	50000−0	24010−0
0.1	1.2589	45662−0	25324−0	45767−0	24840−0	45871−0	24360−0	45972−0	23883−0
0.2	1.5849	41396−0	24874−0	41602−0	24417−0	41803−0	23962−0	42002−0	23508−0
0.3	1.9953	37270−0	24151−0	37566−0	23735−0	37857−0	23319−0	38144−0	22903−0
0.4	2.5119	33342−0	23189−0	33715−0	22825−0	34083−0	22460−0	34446−0	22093−0
0.5	3.1623	29656−0	22032−0	30091−0	21729−0	30521−0	21422−0	30947−0	21111−0
0.6	3.9811	26244−0	20728−0	26724−0	20490−0	27201−0	20245−0	27675−0	19994−0
0.7	5.0119	23124−0	19327−0	23633−0	19152−0	24141−0	18970−0	24648−0	18780−0
0.8	6.3096	20300−0	17872−0	20824−0	17759−0	21349−0	17637−0	21875−0	17506−0
0.9	7.9433	17768−0	16406−0	18294−0	16348−0	18824−0	16282−0	19356−0	16206−0
1.0	10.000	15514−0	14960−0	16032−0	14952−0	16556−0	14935−0	17085−0	14908−0
1.1	12.589	13521−0	13562−0	14023−0	13596−0	14533−0	13621−0	15050−0	13637−0
1.2	15.849	11768−0	12232−0	12247−0	12299−0	12737−0	12359−0	13235−0	12411−0
1.3	19.953	10231−0	10981−0	10685−0	11076−0	11150−0	11164−0	11625−0	11245−0
1.4	25.119	88892−1	98200−1	93142−1	99352−1	97514−1	10045−0	10201−0	10148−0
1.5	31.623	77198−1	87511−1	81150−1	88807−1	85231−1	90058−1	89443−1	91262−1
1.6	39.811	67027−1	77750−1	70677−1	79139−1	74463−1	80496−1	78386−1	81817−1
1.7	50.119	58190−1	68897−1	61545−1	70336−1	65037−1	71755−1	68670−1	73150−1
1.8	63.096	50519−1	60914−1	53589−1	62368−1	56796−1	63813−1	60145−1	65244−1
1.9	79.433	43864−1	53749−1	46662−1	55191−1	49596−1	56633−1	52671−1	58072−1
2.0	100.00	38091−1	47345−1	40633−1	48754−1	43309−1	50171−1	46123−1	51593−1
2.1	125.89	33084−1	41643−1	35388−1	43002−1	37821−1	44376−1	40388−1	45763−1
2.2	158.49	28741−1	36580−1	30824−1	37878−1	33032−1	39197−1	35368−1	40534−1
2.3	199.53	24974−1	32097−1	26854−1	33326−1	28852−1	34581−1	30974−1	35859−1
2.4	251.19	21704−1	28136−1	23398−1	29292−1	25204−1	30476−1	27128−1	31689−1
2.5	316.23	18867−1	24643−1	20391−1	25723−1	22020−1	26835−1	23761−1	27977−1
2.6	398.11	16404−1	21568−1	17773−1	22572−1	19241−1	23610−1	20814−1	24680−1
2.7	501.19	14265−1	18864−1	15494−1	19794−1	16815−1	20758−1	18235−1	21755−1
2.8	630.96	12407−1	16491−1	13508−1	17348−1	14696−1	18239−1	15977−1	19165−1
2.9	794.33	10793−1	14409−1	11779−1	15196−1	12846−1	16018−1	13999−1	16874−1
3.0	1000.0	93897−2	12584−1	10273−1	13305−1	11230−1	14060−1	12268−1	14849−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.56$		$\beta=0.55$		$\beta=0.54$		$\beta=0.53$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	23528−0	50000−0	23050−0	50000−0	22576−0	50000−0	22105−0
0.1	1.2589	46072−0	23409−0	46169−0	22939−0	46266−0	22471−0	46360−0	22007−0
0.2	1.5849	42197−0	23057−0	42389−0	22608−0	42577−0	22161−0	42762−0	21716−0
0.3	1.9953	38426−0	22488−0	38704−0	22073−0	38978−0	21658−0	39248−0	21244−0
0.4	2.5119	34805−0	21724−0	35158−0	21353−0	35508−0	20981−0	35852−0	20608−0
0.5	3.1623	31368−0	20796−0	31786−0	20477−0	32198−0	20155−0	32607−0	19829−0
0.6	3.9811	28145−0	19737−0	28612−0	19475−0	29076−0	19207−0	29536−0	18933−0
0.7	5.0119	25153−0	18583−0	25656−0	18378−0	26158−0	18166−0	26657−0	17947−0
0.8	6.3096	22401−0	17367−0	22928−0	17219−0	23454−0	17062−0	23981−0	16897−0
0.9	7.9433	19891−0	16120−0	20429−0	16026−0	20969−0	15922−0	21511−0	15809−0
1.0	10.000	17619−0	14872−0	18158−0	14826−0	18701−0	14771−0	19249−0	14706−0
1.1	12.589	15574−0	13644−0	16105−0	13641−0	16643−0	13629−0	17188−0	13607−0
1.2	15.849	13743−0	12455−0	14260−0	12489−0	14785−0	12515−0	15320−0	12531−0
1.3	19.953	12111−0	11318−0	12608−0	11384−0	13116−0	11441−0	13634−0	11490−0
1.4	25.119	10662−0	10245−0	11136−0	10336−0	11621−0	10419−0	12119−0	10494−0
1.5	31.623	93787−1	92413−1	98263−1	93508−1	10287−0	94543−1	10761−0	95514−1
1.6	39.811	82447−1	83098−1	86649−1	84335−1	90994−1	85524−1	95481−1	86660−1
1.7	50.119	72447−1	74516−1	76369−1	75851−1	80439−1	77149−1	84660−1	78407−1
1.8	63.096	63639−1	66660−1	67282−1	68054−1	71076−1	69423−1	75026−1	70763−1
1.9	79.433	55891−1	59504−1	59259−1	60926−1	62782−1	62334−1	66462−1	63723−1
2.0	100.00	49079−1	53017−1	52184−1	54441−1	55441−1	55860−1	58856−1	57271−1
2.1	125.89	43095−1	47160−1	45947−1	48564−1	48950−1	49972−1	52109−1	51381−1
2.2	158.49	37840−1	41889−1	40453−1	43257−1	43213−1	44638−1	46127−1	46026−1
2.3	199.53	33226−1	37160−1	35614−1	38480−1	38146−1	39819−1	40826−1	41173−1
2.4	251.19	29176−1	32928−1	31354−1	34191−1	33671−1	35478−1	36131−1	36786−1
2.5	316.23	25620−1	29149−1	27604−1	30350−1	29720−1	31578−1	31974−1	32831−1
2.6	398.11	22500−1	25782−1	24303−1	26916−1	26232−1	28080−1	28295−1	29274−1
2.7	501.19	19760−1	22787−1	21398−1	23852−1	23154−1	24949−1	25038−1	26080−1
2.8	630.96	17356−1	20126−1	18841−1	21122−1	20438−1	22152−1	22155−1	23217−1
2.9	794.33	15245−1	17766−1	16590−1	18693−1	18041−1	19656−1	19605−1	20654−1
3.0	1000.0	13392−1	15674−1	14608−1	16534−1	15925−1	17431−1	17348−1	18364−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.52$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.52$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.51$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.51$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.50$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.50$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.48$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.48$ $\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	21637−0	50000−0	21172−0	50000−0	20711−0	50000−0	19796−0
0.1	1.2589	46453−0	21545−0	46544−0	21086−0	46634−0	20630−0	46809−0	19727−0
0.2	1.5849	42945−0	21273−0	43124−0	20832−0	43301−0	20393−0	43645−0	19519−0
0.3	1.9953	39513−0	20831−0	39775−0	20417−0	40033−0	20005−0	40538−0	19181−0
0.4	2.5119	36193−0	20233−0	36529−0	19857−0	36861−0	19479−0	37512−0	18721−0
0.5	3.1623	33012−0	19500−0	33412−0	19167−0	33808−0	18832−0	34589−0	18152−0
0.6	3.9811	29993−0	18654−0	30447−0	18370−0	30897−0	18082−0	31788−0	17489−0
0.7	5.0119	27155−0	17721−0	27650−0	17488−0	28144−0	17248−0	29124−0	16750−0
0.8	6.3096	24507−0	16723−0	25033−0	16542−0	25559−0	16352−0	26608−0	15949−0
0.9	7.9433	22056−0	15686−0	22601−0	15555−0	23149−0	15414−0	24247−0	15106−0
1.0	10.000	19801−0	14631−0	20356−0	14546−0	20915−0	14452−0	22044−0	14235−0
1.1	12.589	17738−0	13576−0	18295−0	13535−0	18858−0	13484−0	20000−0	13351−0
1.2	15.849	15862−0	12538−0	16413−0	12535−0	16972−0	12523−0	18112−0	12468−0
1.3	19.953	14163−0	11530−0	14701−0	11561−0	15250−0	11583−0	16376−0	11597−0
1.4	25.119	12629−0	10562−0	13150−0	10621−0	13684−0	10672−0	14785−0	10747−0
1.5	31.623	11249−0	96416−1	11750−0	97246−1	12264−0	97999−1	13333−0	99259−1
1.6	39.811	10011−0	87739−1	10489−0	88757−1	10982−0	89708−1	12010−0	91396−1
1.7	50.119	89034−1	79619−1	93562−1	80782−1	98248−1	81889−1	10809−0	83921−1
1.8	63.096	79134−1	72070−1	83404−1	73338−1	87838−1	74563−1	97208−1	76863−1
1.9	79.433	70303−1	65090−1	74310−1	66430−1	78486−1	67738−1	87360−1	70237−1
2.0	100.00	62434−1	58669−1	66179−1	60051−1	70097−1	61412−1	78466−1	64050−1
2.1	125.89	55430−1	52788−1	58918−1	54187−1	62580−1	55575−1	70443−1	58299−1
2.2	158.49	49200−1	47420−1	52439−1	48816−1	55851−1	50210−1	63215−1	52975−1
2.3	199.53	43663−1	42539−1	46663−1	43915−1	49832−1	45297−1	56709−1	48064−1
2.4	251.19	38744−1	38113−1	41515−1	39455−1	44453−1	40811−1	50858−1	43549−1
2.5	316.23	34375−1	34109−1	36930−1	35408−1	39647−1	36727−1	45600−1	39410−1
2.6	398.11	30497−1	30496−1	32848−1	31744−1	35356−1	33016−1	40876−1	35625−1
2.7	501.19	27055−1	27241−1	29215−1	28433−1	31526−1	29653−1	36636−1	32171−1
2.8	630.96	24001−1	24315−1	25982−1	25447−1	28108−1	26610−1	32831−1	29027−1
2.9	794.33	21291−1	21689−1	23106−1	22758−1	25059−1	23862−1	29417−1	26169−1
3.0	1000.0	18886−1	19334−1	20547−1	20340−1	22339−1	21383−1	26356−1	23575−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.46$		$\beta=0.45$		$\beta=0.44$		$\beta=0.42$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_8}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	18893−0	50000−0	18446−0	50000−0	18001−0	50000−0	17119−0
0.1	1.2589	46978−0	18833−0	47061−0	18390−0	47142−0	17949−0	47302−0	17074−0
0.2	1.5849	43980−0	18653−0	44144−0	18223−0	44305−0	17794−0	44620−0	16941−0
0.3	1.9953	41029−0	18359−0	41270−0	17949−0	41507−0	17540−0	41973−0	16723−0
0.4	2.5119	38147−0	17959−0	38459−0	17576−0	38768−0	17193−0	39374−0	16424−0
0.5	3.1623	35354−0	17461−0	35730−0	17112−0	36103−0	16760−0	36838−0	16050−0
0.6	3.9811	32665−0	16879−0	33098−0	16568−0	33529−0	16252−0	34379−0	15609−0
0.7	5.0119	30095−0	16226−0	30577−0	15956−0	31056−0	15679−0	32007−0	15110−0
0.8	6.3096	27654−0	15516−0	28175−0	15288−0	28695−0	15052−0	29731−0	14561−0
0.9	7.9433	25349−0	14762−0	25901−0	14577−0	26453−0	14383−0	27558−0	13971−0
1.0	10.000	23184−0	13978−0	23758−0	13836−0	24335−0	13684−0	25493−0	13351−0
1.1	12.589	21162−0	13178−0	21750−0	13076−0	22343−0	12964−0	23540−0	12709−0
1.2	15.849	19282−0	12372−0	19877−0	12308−0	20478−0	12234−0	21700−0	12053−0
1.3	19.953	17540−0	11571−0	18135−0	11542−0	18739−0	11503−0	19973−0	11392−0
1.4	25.119	15932−0	10783−0	16523−0	10786−0	17124−0	10779−0	18357−0	10732−0
1.5	31.623	14454−0	10017−0	15034−0	10048−0	15627−0	10069−0	16850−0	10080−0
1.6	39.811	13098−0	92765−1	13664−0	93319−1	14245−0	93781−1	15449−0	94409−1
1.7	50.119	11858−0	85675−1	12407−0	86436−1	12972−0	87112−1	14150−0	88193−1
1.8	63.096	10726−0	78929−1	11255−0	79861−1	11801−0	80719−1	12948−0	82188−1
1.9	79.433	96952−1	72547−1	10202−0	73618−1	10728−0	74624−1	11837−0	76421−1
2.0	100.00	87576−1	66543−1	92419−1	67721−1	97459−1	68846−1	10814−0	70912−1
2.1	125.89	79063−1	60920−1	83669−1	62179−1	88479−1	63395−1	98729−1	65676−1
2.2	158.49	71343−1	55677−1	75708−1	56992−1	80283−1	58274−1	90081−1	60720−1
2.3	199.53	64350−1	50806−1	68474−1	52155−1	72810−1	53482−1	82146−1	56048−1
2.4	251.19	58021−1	46295−1	61907−1	47660−1	66006−1	49013−1	74874−1	51659−1
2.5	316.23	52299−1	42131−1	55950−1	43497−1	59814−1	44858−1	68216−1	47550−1
2.6	398.11	47128−1	38298−1	50552−1	39650−1	54186−1	41007−1	62127−1	43714−1
2.7	501.19	42459−1	34777−1	45662−1	36105−1	49074−1	37445−1	56562−1	40143−1
2.8	630.96	38245−1	31550−1	41237−1	32846−1	44432−1	34159−1	51481−1	36825−1
2.9	794.33	34443−1	28598−1	37232−1	29854−1	40221−1	31133−1	46843−1	33750−1
3.0	1000.0	31014−1	25903−1	33611−1	27114−1	36402−1	28353−1	42613−1	30905−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.40$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.40$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.38$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.38$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.36$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.36$ $\frac{\bar{C}''}{C_0-C_\infty}$	$\beta=0.35$ $\frac{\bar{C}'}{C_0-C_\infty}$	$\beta=0.35$ $\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	16246−0	50000−0	15382−0	50000−0	14526−0	50000−0	14101−0
0.1	1.2589	47457−0	16208−0	47607−0	15350−0	47754−0	14499−0	47826−0	14077−0
0.2	1.5849	44928−0	16095−0	45227−0	15254−0	45518−0	14419−0	45662−0	14003−0
0.3	1.9953	42426−0	15908−0	42869−0	15096−0	43301−0	14286−0	43514−0	13881−0
0.4	2.5119	39966−0	15652−0	40546−0	14878−0	41113−0	14102−0	41392−0	13714−0
0.5	3.1623	37559−0	15331−0	38266−0	14605−0	38961−0	13871−0	39303−0	13502−0
0.6	3.9811	35216−0	14952−0	36041−0	14280−0	36854−0	13596−0	37255−0	13249−0
0.7	5.0119	32948−0	14519−0	33878−0	13909−0	34799−0	13280−0	35255−0	12959−0
0.8	6.3096	30761−0	14042−0	31785−0	13497−0	32802−0	12928−0	33307−0	12634−0
0.9	7.9433	28663−0	13527−0	29767−0	13050−0	30869−0	12544−0	31419−0	12280−0
1.0	10.000	26659−0	12981−0	27830−0	12575−0	29004−0	12134−0	29593−0	11900−0
1.1	12.589	24752−0	12413−0	25977−0	12077−0	27212−0	11701−0	27833−0	11499−0
1.2	15.849	22945−0	11829−0	24210−0	11561−0	25494−0	11251−0	26143−0	11080−0
1.3	19.953	21238−0	11236−0	22532−0	11035−0	23854−0	10788−0	24524−0	10648−0
1.4	25.119	19631−0	10640−0	20943−0	10502−0	22290−0	10317−0	22977−0	10207−0
1.5	31.623	18122−0	10047−0	19441−0	99684−1	20805−0	98417−1	21503−0	97600−1
1.6	39.811	16711−0	94617−1	18027−0	94375−1	19398−0	93657−1	20103−0	93112−1
1.7	50.119	15392−0	88880−1	16699−0	89135−1	18067−0	88925−1	18775−0	88636−1
1.8	63.096	14165−0	83295−1	15453−0	83996−1	16812−0	84251−1	17518−0	84199−1
1.9	79.433	13024−0	77891−1	14288−0	78987−1	15630−0	79662−1	16331−0	79827−1
2.0	100.00	11965−0	72691−1	13200−0	74131−1	14520−0	75180−1	15212−0	75542−1
2.1	125.89	10985−0	67710−1	12186−0	69445−1	13478−0	70823−1	14159−0	71361−1
2.2	158.49	10078−0	62961−1	11242−0	64944−1	12502−0	66608−1	13170−0	67302−1
2.3	199.53	92408−1	58452−1	10365−0	60638−1	11590−0	62545−1	12242−0	63375−1
2.4	251.19	84687−1	54185−1	95504−1	56534−1	10738−0	58645−1	11373−0	59591−1
2.5	316.23	77574−1	50161−1	87957−1	52635−1	99430−1	54912−1	10559−0	55957−1
2.6	398.11	71029−1	46377−1	80969−1	48943−1	92024−1	51352−1	97992−1	52478−1
2.7	501.19	65010−1	42829−1	74504−1	45456−1	85131−1	47966−1	90894−1	49157−1
2.8	630.96	59482−1	39511−1	68530−1	42172−1	78721−1	44754−1	84275−1	45995−1
2.9	794.33	54406−1	36414−1	63012−1	39086−1	72766−1	41714−1	78106−1	42992−1
3.0	1000.0	49750−1	33530−1	57920−1	36192−1	67237−1	38844−1	72362−1	40147−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.34$		$\beta=0.32$		$\beta=0.30$		$\beta=0.25$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	13678−0	50000−0	12838−0	50000−0	12004−0	50000−0	99456−1
0.1	1.2589	47898−0	13656−0	48038−0	12819−0	48174−0	11989−0	48504−0	99371−1
0.2	1.5849	45803−0	13589−0	46081−0	12764−0	46354−0	11944−0	47012−0	99114−1
0.3	1.9953	43724−0	13478−0	44138−0	12672−0	44543−0	11869−0	45524−0	98689−1
0.4	2.5119	41668−0	13324−0	42213−0	12545−0	42748−0	11765−0	44045−0	98098−1
0.5	3.1623	39643−0	13131−0	40313−0	12385−0	40973−0	11633−0	42577−0	97346−1
0.6	3.9811	37654−0	12899−0	38443−0	12192−0	39221−0	11475−0	41122−0	96437−1
0.7	5.0119	35709−0	12633−0	36608−0	11970−0	37499−0	11292−0	39683−0	95377−1
0.8	6.3096	33811−0	12335−0	34814−0	11721−0	35808−0	11086−0	38262−0	94173−1
0.9	7.9433	31968−0	12009−0	33063−0	11447−0	34154−0	10858−0	36862−0	92834−1
1.0	10.000	30182−0	11658−0	31360−0	11150−0	32540−0	10611−0	35484−0	91368−1
1.1	12.589	28457−0	11287−0	29709−0	10835−0	30967−0	10347−0	34131−0	89783−1
1.2	15.849	26795−0	10898−0	28111−0	10503−0	29439−0	10068−0	32804−0	88090−1
1.3	19.953	25200−0	10496−0	26569−0	10158−0	27958−0	97756−1	31504−0	86299−1
1.4	25.119	23672−0	10084−0	25084−0	98027−1	26525−0	94730−1	30234−0	84419−1
1.5	31.623	22212−0	96657−1	23658−0	94393−1	25142−0	91619−1	28994−0	82460−1
1.6	39.811	20820−0	92440−1	22291−0	90707−1	23809−0	88444−1	27786−0	80434−1
1.7	50.119	19497−0	88220−1	20984−0	86993−1	22528−0	85225−1	26610−0	78349−1
1.8	63.096	18240−0	84022−1	19736−0	83275−1	21297−0	81981−1	25467−0	76217−1
1.9	79.433	17051−0	79871−1	18547−0	79572−1	20119−0	78729−1	24358−0	74047−1
2.0	100.00	15926−0	75787−1	17416−0	75904−1	18991−0	75486−1	23283−0	71848−1
2.1	125.89	14864−0	71790−1	16342−0	72288−1	17914−0	72266−1	22242−0	69630−1
2.2	158.49	13863−0	67893−1	15324−0	68738−1	16887−0	69083−1	21236−0	67401−1
2.3	199.53	12921−0	64110−1	14360−0	65267−1	15909−0	65949−1	20264−0	65169−1
2.4	251.19	12036−0	60452−1	13449−0	61885−1	14979−0	62874−1	19327−0	62941−1
2.5	316.23	11205−0	56926−1	12588−0	58603−1	14095−0	59868−1	18423−0	60725−1
2.6	398.11	10427−0	53538−1	11776−0	55426−1	13256−0	56937−1	17554−0	58528−1
2.7	501.19	96972−1	50292−1	11011−0	52360−1	12461−0	54089−1	16718−0	56355−1
2.8	630.96	90150−1	47191−1	10291−0	49410−1	11708−0	51328−1	15914−0	54211−1
2.9	794.33	83773−1	44235−1	96137−1	46578−1	10996−0	48659−1	15143−0	52101−1
3.0	1000.0	77817−1	41425−1	89776−1	43866−1	10323−0	46085−1	14403−0	50031−1

$\log_{10}\omega\tau$	$\omega\tau$	$\beta=0.20$		$\beta=0.15$		$\beta=0.10$		$\beta=0.05$	
		$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$	$\frac{\bar{C}'}{C_0-C_\infty}$	$\frac{\bar{C}''}{C_0-C_\infty}$
0.0	1.0000	50000−0	79192−1	50000−0	59179−1	50000−0	39351−1	50000−0	19645−1
0.1	1.2589	48820−0	79149−1	49125−0	59161−1	49421−0	39346−1	49712−0	19644−1
0.2	1.5849	47641−0	79020−1	48250−0	59107−1	48842−0	39330−1	49423−0	19642−1
0.3	1.9953	46466−0	78806−1	47376−0	59018−1	48263−0	39304−1	49135−0	19639−1
0.4	2.5119	45294−0	78508−1	46504−0	58893−1	47685−0	39267−1	48847−0	19635−1
0.5	3.1623	44127−0	78126−1	45633−0	58734−1	47107−0	39220−1	48559−0	19629−1
0.6	3.9811	42967−0	77663−1	44766−0	58539−1	46530−0	39163−1	48271−0	19622−1
0.7	5.0119	41815−0	77121−1	43902−0	58311−1	45954−0	39095−1	47983−0	19613−1
0.8	6.3096	40672−0	76501−1	43042−0	58048−1	45380−0	39017−1	47695−0	19603−1
0.9	7.9433	39539−0	75807−1	42185−0	57753−1	44806−0	38929−1	47408−0	19592−1
1.0	10.000	38417−0	75041−1	41334−0	57425−1	44234−0	38831−1	47121−0	19580−1
1.1	12.589	37308−0	74207−1	40488−0	57066−1	43663−0	38723−1	46833−0	19566−1
1.2	15.849	36213−0	73307−1	39647−0	56675−1	43094−0	38605−1	46546−0	19551−1
1.3	19.953	35132−0	72346−1	38813−0	56255−1	42527−0	38477−1	46260−0	19535−1
1.4	25.119	34066−0	71327−1	37985−0	55806−1	41962−0	38340−1	45973−0	19518−1
1.5	31.623	33016−0	70254−1	37164−0	55329−1	41399−0	38193−1	45687−0	19499−1
1.6	39.811	31984−0	69131−1	36351−0	54825−1	40838−0	38037−1	45401−0	19479−1
1.7	50.119	30969−0	67961−1	35545−0	54296−1	40279−0	37872−1	45115−0	19458−1
1.8	63.096	29972−0	66750−1	34748−0	53741−1	39724−0	37698−1	44830−0	19435−1
1.9	79.433	28995−0	65500−1	33959−0	53164−1	39170−0	37516−1	44545−0	19412−1
2.0	100.00	28037−0	64217−1	33179−0	52564−1	38620−0	37324−1	44260−0	19387−1
2.1	125.89	27099−0	62903−1	32409−0	51943−1	38072−0	37124−1	43976−0	19360−1
2.2	158.49	26181−0	61564−1	31648−0	51302−1	37528−0	36916−1	43692−0	19333−1
2.3	199.53	25284−0	60203−1	30896−0	50642−1	36986−0	36700−1	43408−0	19304−1
2.4	251.19	24408−0	58825−1	30155−0	49966−1	36448−0	36477−1	43125−0	19274−1
2.5	316.23	23553−0	57432−1	29424−0	49273−1	35914−0	36245−1	42843−0	19243−1
2.6	398.11	22719−0	56028−1	28704−0	48565−1	35382−0	36006−1	42561−0	19211−1
2.7	501.19	21907−0	54617−1	27994−0	47844−1	34855−0	35761−1	42279−0	19177−1
2.8	630.96	21116−0	53203−1	27296−0	47111−1	34331−0	35508−1	41998−0	19143−1
2.9	794.33	20347−0	51787−1	26608−0	46367−1	33811−0	35248−1	41717−0	19107−1
3.0	1000.0	19598−0	50375−1	25932−0	45613−1	33295−0	34982−1	41437−0	19070−1

Table 2. Numerical Data of Real and Imaginary Part of the Complex Conductance for the Circular Arc Rule.

$\ln\omega\tau$	$\omega\tau$	$\beta=1.00$		$\beta=0.90$		$\beta=0.80$	
		$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$	$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$	$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$
−4.5	0.01111	0.00012	0.01111	0.00019	0.01108	0.00028	0.01101
−4.0	0.01832	0.00034	0.01831	0.00049	0.01823	0.00069	0.01807
−3.5	0.03020	0.00091	0.03017	0.00126	0.02994	0.00168	0.02954
−3.0	0.04979	0.00247	0.04966	0.00322	0.04906	0.00404	0.04809
−2.5	0.08209	0.00669	0.08154	0.00818	0.07992	0.00959	0.07761
−2.0	0.1353	0.01799	0.1329	0.02048	0.1287	0.02230	0.1234
−1.5	0.2231	0.04743	0.2126	0.04975	0.2022	0.05006	0.1910
−1.0	0.3679	0.1192	0.3240	0.1143	0.3027	0.1063	0.2832
−0.5	0.6065	0.2689	0.4434	0.2378	0.4153	0.2075	0.3929
−0.25	0.7788	0.3775	0.4848	0.3254	0.4642	0.2786	0.4484
+0.00	1.000	0.5000	0.5000	0.4270	0.5000	0.3633	0.5000
+0.25	1.284	0.6225	0.4848	0.5365	0.5188	0.4594	0.5448
+0.5	1.649	0.7311	0.4434	0.6465	0.5198	0.5640	0.5808
+1.0	2.718	0.8808	0.3240	0.8445	0.4814	0.7851	0.6260
+1.5	4.482	0.9526	0.2126	0.9993	0.4206	1.005	0.6451
+2.0	7.389	0.9820	0.1329	1.118	0.3642	1.217	0.6540
+2.5	12.18	0.9933	0.08154	1.215	0.3220	1.423	0.6649
+3.0	20.09	0.9975	0.04966	1.300	0.2944	1.628	0.6844
+3.5	33.12	0.9991	0.03017	1.381	0.2786	1.839	0.7152
+4.0	54.60	0.9997	0.01831	1.460	0.2716	2.061	0.7581
+4.5	90.02	0.9999	0.01111	1.540	0.2711	2.299	0.8129
+5.0	148.4	0.9999	0.00672	1.623	0.2752		
+6.0	403.4			1.797	0.2928		
+7.0	1096.6			1.988	0.3185		

$\ln\omega\tau$	$\omega\tau$	$\beta=0.70$		$\beta=0.60$		$\beta=0.40$		$\beta=0.20$	
		$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$	$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$	$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$	$\frac{\bar{G}'\tau}{C_0-C_\infty}$	$\frac{\bar{G}''\tau}{C_0-C_\infty}$
−4.5	0.01111	0.00041	0.01088	0.00056	0.01066	0.00083	0.00973	0.00072	0.00795
−4.0	0.01832	0.00094	0.01778	0.00121	0.01731	0.00159	0.01559	0.00124	0.01272
−3.5	0.03020	0.00214	0.02890	0.00258	0.02793	0.00300	0.02481	0.00212	0.02029
−3.0	0.04979	0.00482	0.04667	0.00545	0.04471	0.00559	0.03924	0.00360	0.03231
−2.5	0.08209	0.01070	0.07455	0.01129	0.07077	0.01025	0.06155	0.00610	0.05133
−2.0	0.1353	0.02315	0.1171	0.02283	0.1103	0.01853	0.09567	0.01029	0.08135
−1.5	0.2231	0.04831	0.1795	0.04466	0.1682	0.03288	0.1472	0.01727	0.1286
−1.0	0.3679	0.09589	0.2656	0.08392	0.2500	0.05720	0.2239	0.02884	0.2027
−0.5	0.6065	0.1783	0.3747	0.1502	0.3598	0.09746	0.3367	0.04791	0.3188
−0.25	0.7788	0.2361	0.4360	0.1970	0.4261	0.1262	0.4109	0.06164	0.3994
+0.00	1.000	0.3064	0.5000	0.2548	0.5000	0.1625	0.5000	0.07919	0.5000
+0.25	1.284	0.3893	0.5652	0.3248	0.5816	0.2080	0.6065	0.1016	0.6255
+0.5	1.649	0.4845	0.6301	0.4084	0.6706	0.2649	0.7336	0.1302	0.7821
+1.0	2.718	0.7085	0.7559	0.6201	0.8712	0.4227	1.064	0.2131	1.220
+1.5	4.482	0.9703	0.8754	0.8971	1.103	0.6604	1.526	0.3469	1.899
+2.0	7.389	1.264	0.9937	1.246	1.369	1.012	2.166	0.5618	2.948
+2.5	12.18	1.588	1.119	1.676	1.680	1.522	3.048	0.9056	4.565
+3.0	20.09	1.946	1.259	2.199	2.047	2.253	4.256	1.453	7.050
+3.5	33.12	2.345	1.422	2.831	2.485	3.288	5.905	2.319	10.86
+4.0	54.60	2.794	1.614	3.594	3.014	4.738	8.149	3.686	16.71
+4.5	90.02	3.302	1.842	4.517	3.657	6.755	11.20	5.834	25.63

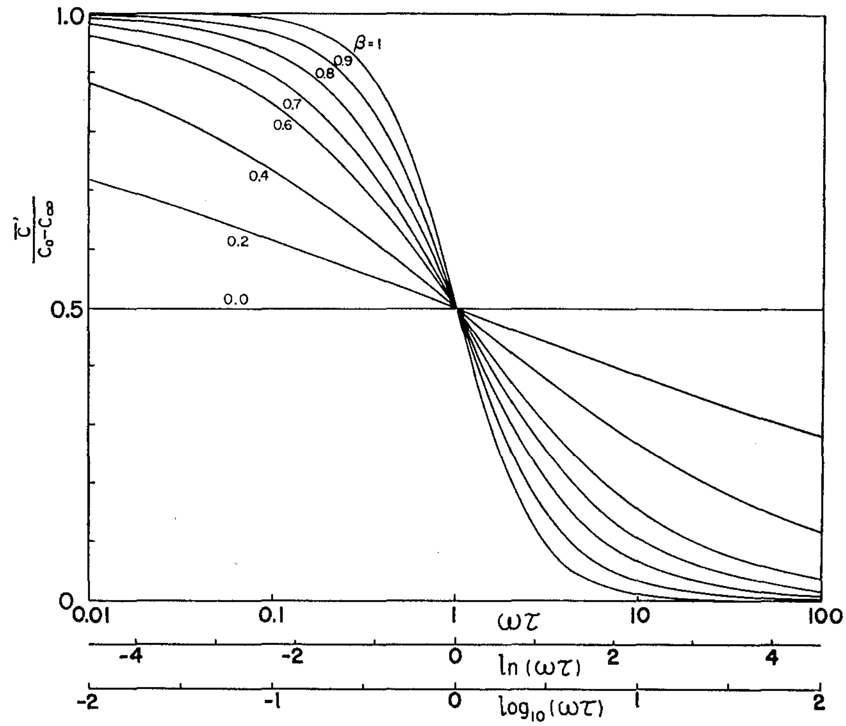

Fig. 1. The real part of the complex capacitance for the circular arc rule as a function of frequency. Numbers beside the curves denote the values of β.

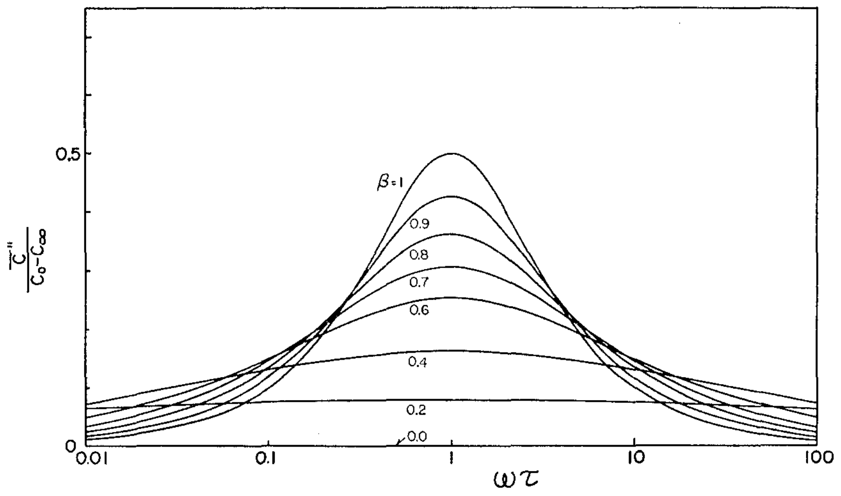

Fig. 2. The imaginary part of the complex capacitance for the circular arc rule as a function of frequency. Numbers beside the curves denote the values of β.

IV. DISCUSSIONS

A. The Real and the Imaginary Part of the Complex Capacitance

The behavior of the complex capacitance given in Table 1 and shown in

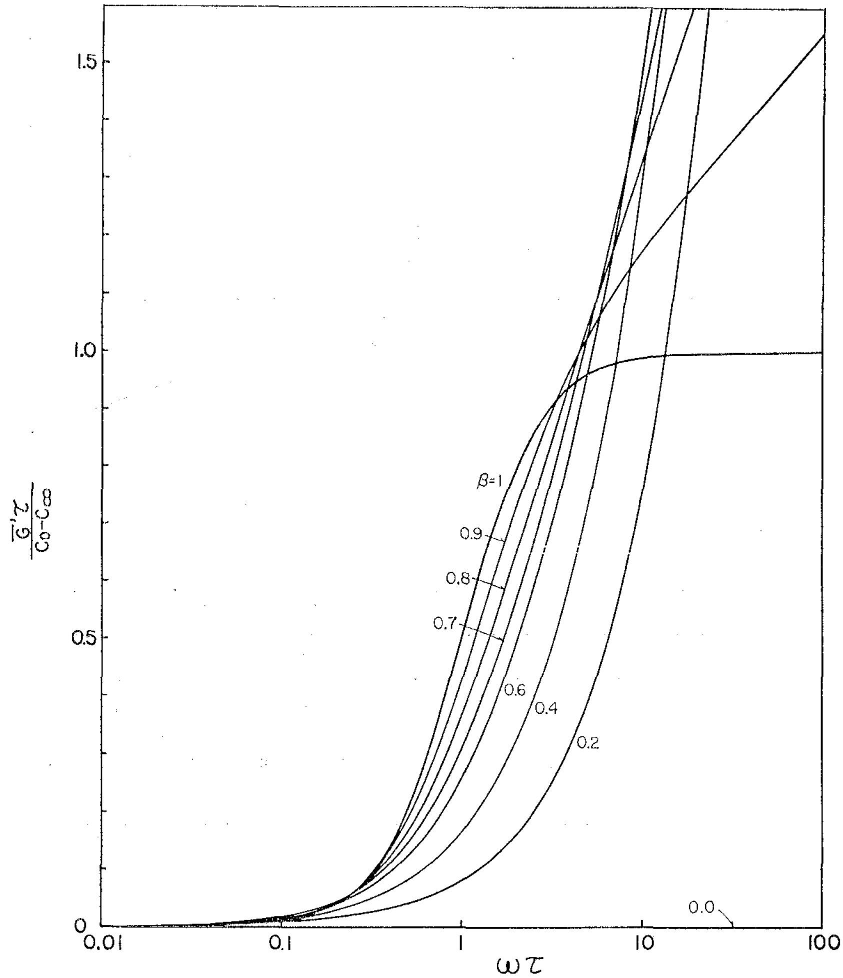

Fig. 3. The real part of the complex conductance for the circular arc rule as a function of frequency. Numbers beside the curves denote the value of β.

Figs. 1, 2 and 5 is the same as has been known so far. The numerical data of complex capacitance are useful for accurate determination of the distribution parameter β in the system with a wide distribution of relaxation times[5] and also for facilitating the analysis of the system with multiple relaxation regions.[6]

B. The Frequency Dependence of the Real and the Imaginary Part of the Complex Conductance

As seen in Fig. 3, the real part G' of the complex conductance for $\beta=1$ increases markedly in the $\omega\tau$-range of 0.1 to 10, and tends to $(C_0-C_\infty)/\tau$ at higher frequencies. In the case of $\beta<1$, the conductance increases infinitely at higher frequencies.

The frequency dependence of the imaginary part G'' of the complex conductance, which has been discussed very little so far, is seen in Fig. 4. In the

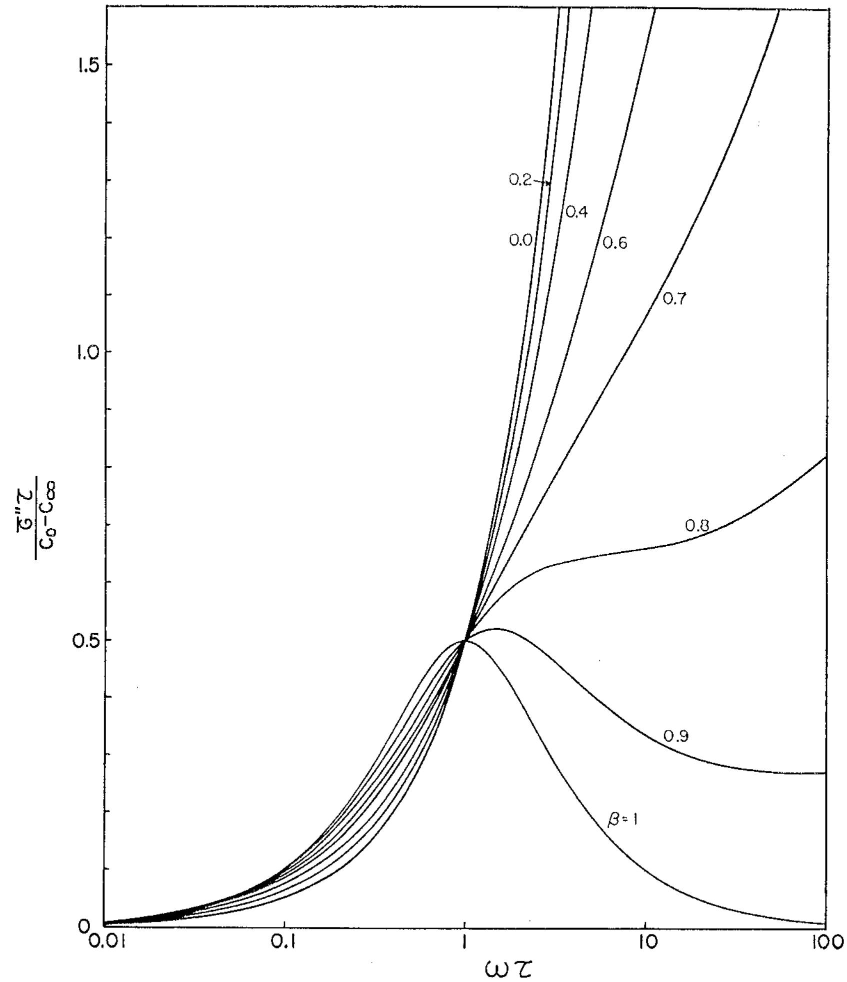

Fig. 4. The imaginary part of the complex conductance for the circular arc rule as a function of frequency. Numbers beside the curves denote the value of β.

case of $\beta=1$, G'' attains its maximum value $(C_0-C_\infty)/2\tau$ at $\omega\tau=1$, and decreases monotonously to zero at higher frequencies. The curve of G'' against $\ln(\omega\tau)$ is symmetrical with respect to an axis $\omega\tau=1$. In the case of $\beta<1$, it is worthy of being noted that G'' does not tend to zero at the higher frequency side. For $\beta=0.9$, G'' shows a maximum value at $\omega\tau=1.46$, then decreases temporarily to a certain minimum value with increasing frequency, and finally increases infinitely at the higher frequency side. For $\beta<0.810$, G'' increases with increasing frequency over the whole range of frequency.

C. The Complex Plane Plots of Conductance

Figure 6 shows the complex plane plots of the complex conductance for respective values of β. The plot for $\beta=1$ gives a semicircle. For $\beta<1$ the complex conductance plots deviate from a semicircle. For $\beta=0.9$ the imaginary part of the admittance plot shows a maximum value at the frequency given by $\omega\tau=$

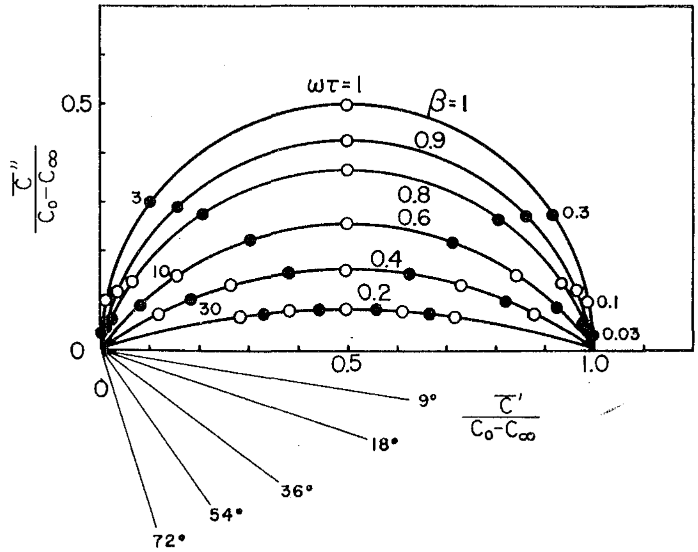

Fig. 5. Complex plane plots of the complex capacitance for the circular arc rule. Numbers beside the curves indicate the values of $\omega\tau$ and β.

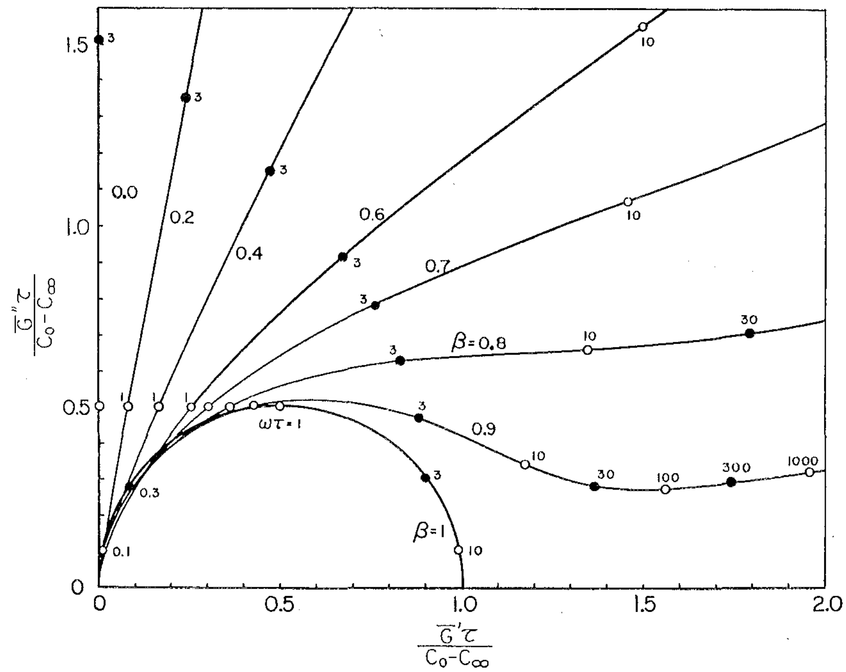

Fig. 6. Complex plane plots of the complex conductance associated with the circular arc rule. Numbers beside the curves indicate the values of $\omega\tau$ and β.

1.46. At higher frequencies the plots deviate markedly from a semicircle. In the case of $\beta<0.810$, $\bar{G}''$ increases monotonously with the increase of $\bar{G}'$, resulting in the remarkable deviation of complex conductance plots from a semicircle or a circular arc.

D. Some Comments on the Analysis of the Data Characterized by the Circular Arc Rule

It is readily seen in Fig. 3 that the conductance increases infinitely at higher frequencies so far as $\beta<1$ which corresponds to the cases of circular arcs.

Pauly *et al.* reported the dielectric dispersions of suspensions of guinea pig heart mitochondria[3] and of rat liver mitochondria.[4] The main dispersion of the dielectric constant observed by them gave a circular arc in the complex conductance plane, and were understood reasonably to be attributed to the heterogeneous structure characteristic of a suspension of mitochondrial globules covered with less conductive membranes. Moreover they observed a further rise of the conductivity at the frequencies about ten times as high as the critical frequency of the main dispersion. They suggested that this rise of the conductivity is the beginning of another dielectric dispersion due to the heterogeneous structure inside the mitochondrial globules. According to Fig. 3, however, such a rise of the conductivity at higher frequencies ($\omega\tau>10$) is readily interpretable as the property concomitant with the circular arc rule without any other dispersion mechanism.

In the papers aforementioned[3,4] the critical frequency for the conductivity or the conductance dispersion was not identical with that for the dielectric or the capacitance dispersion. In view of this discrepancy Pauly *et al.* suggested that a reasonable approximation for the experimental determination of the critical frequency f_0 may be the geometrical mean of $f_{0\varepsilon}$ and $f_{0\kappa}$, that is,

$$f_0=\sqrt{f_{0\varepsilon}\cdot f_{0\kappa}} \tag{28}$$

where $f_{0\varepsilon}$ and $f_{0\kappa}$ are the frequencies at which the dielectric constant $\varepsilon(f)$ and the conductivity $\kappa(f)$ are given by

$$\frac{\varepsilon(f_{0\varepsilon})-\varepsilon_\infty}{\varepsilon_0-\varepsilon_\infty}=\frac{1}{2} \tag{29}$$

and

$$\frac{\kappa(f_{0\kappa})-\kappa_0}{\kappa_\infty-\kappa_0}=\frac{1}{2} \tag{30}$$

respectively.

In the case of the circular arc rule, however, κ_∞ does not necessarily take finite values at high frequencies as seen in Fig. 3. It thus turns out impossible to determine the value of $f_{0\kappa}$ by means of Eq. 30 and to evaluate f_0 from Eq. 28. For the systems showing a circular arc it is concluded that the critical frequency f_0 should be taken to be identical with $f_{0\varepsilon}$ determined by Eq. 29 regardless of the conductivity dispersion.

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